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A Note on Euclidean Wormhole

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Abstract: The Big Bang theory is one of the most successful models for describing the evolution of the universe. However, this theory fails to describe phenomena of the very early universe, precisely near the Planck epoch. It cannot avoid initial singularity. To describe phenomena of the very early universe, quantum gravity must be evoked. The Wheeler-DeWitt (WDW) equation is the quantum counterpart of the classical gravitation field equation. To solve the WDW equation, we need some specific boundary condition. The Hartle-Hawking no boundary proposal is widely used for this purpose. It insists that the time near and beyond the Planck epoch is Euclidean in nature. Classical Euclidean wormholes are solutions of the field equation with Euclidean time, and quantum Euclidean wormholes are solutions of the WDW equation in the Euclidean regime. Euclidean wormholes provide a compelling theoretical tool for probing the deep structure of space-time and quantum gravity. Studying Euclidean wormholes is crucial in theoretical physics because they act as non-perturbative gravitational instantons that provide essential insights into quantum gravity, black hole information, and the structure of space-time at the Planck scale. These geometries, which connect disconnected boundaries or distant regions of space, are instrumental in addressing the black hole information paradox by preserving the unitarity of quantum mechanics. Thus, the study of quantum Euclidean wormholes will help us to understand the very early universe in a better way. In this review, I have discussed both the classical and quantum Euclidean wormholes in a simplified way. The results are quite fascinating. In the case of quantum Euclidean wormholes, some authors have shown that the universe has evolved from Euclidean space-time in the very early universe to late-stage Lorentzian space-time.

Key-words: Early Universe, Wheeler DeWitt Equation, Hawking-Page boundary conditions, Classical and Quantum Euclidean wormholes.

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1. Introduction

Wormholes are theoretical solutions arising from Einstein's General Theory of Relativity (GTR). In GTR, mass and energy curve spacetime. Under certain extreme conditions, space-time might bend so much that it forms a “tunnel” between two regions. Such a type of tunnel is known as a wormhole. It acts like a shortcut, connecting two distant points of the universe. This concept was first explored by Einstein and Rosen in 1935, leading to the idea of the Einstein–Rosen bridge (Einstein et al. 1935). Since its inception, the concept of wormholes has attracted both scientists and the general public, because of its relation with time travel. Instead of travelling through ordinary spacetime, an object could pass through a wormhole and emerge somewhere else much faster. Moreover, due to high speed or strong gravity, it could create a time difference between the two ends. Thus, a traveller entering one mouth of a wormhole may emerge from the other in either the past or the future.

The scientific community became interested in wormholes again during the late 1980s (Giddings et al., 1988; Lavrelashvili et al., 1987; Lavrelashvili et al., 1988; Lavrelashvili et al., 1988). A wormhole connects two asymptotically flat De-Sitter spaces by a `throat'. It acts as a tunnel with two ends, each being located at separate points in spacetime. There are two main types of wormholes: `Lorentzian wormholes' and `Euclidean wormholes'. Lorentzian wormholes are solutions of Einstein's field equations that exist in Lorentzian space-time, and they are essentially shortcuts through space and time (Visser, 1995; Morris et al., 1988). Depending on time travel, they are categorized as non-traversable and traversable wormholes. As their name suggests, non-traversable wormholes collapse too rapidly for any object to pass through them. In contrast, a Lorentzian wormhole that allows matter or observers to pass through without collapsing is known as a traversable wormhole (Morris et al., 1988; Morris et al., 1988).

Detail studies suggest that traversable Lorentzian wormholes should be made of matter that violates the 'Null Energy Condition' (NEC)[1] (Novikov et al., 2007; Shatskii et al. 2008; Hochberg et al., 1997; Hochberg et al., 1998; Visser et al., 2003; Kar et al, 2007). Consequently, traversable Lorentzian wormholes require large amounts of exotic matter[2] with negative energy density to remain open and stable during traversal. However, ordinary forms of matter and energy do not exhibit this property. Small amounts of negative energy can be generated experimentally through the Casimir effect. However, constructing a traversable wormhole would require enormous quantities of such exotic matter, which is far beyond current technological capabilities. Euclidean wormholes are theoretical, non-traversable geometries that arise in Euclidean spacetime, where time is treated as an imaginary coordinate. They act as gravitational instantons connecting distant regions of spacetime or even different universes (Hawking, 1977). Classical Euclidean wormholes are solutions of the Euclidean field equations that connect two asymptotically flat regions through a narrow throat (Hawking, 1979).

In contrast, quantum Euclidean wormholes represent quantum gravitational transitions. They may influence particle physics by renormalizing coupling constants and inducing the nucleation of "baby universes" (Hebecker et al., 2018; Arkani-Hamed et al. 2007; Marolf et al., 2025; Anabalón et al. 2024). Euclidean wormholes correspond to saddle points of the Euclidean gravitational path integral (Wands, 1994; Marolf, 2022). They are hypothetical spacetime configurations that arise naturally in Euclidean quantum gravity. In the Euclidean regime, spacetime is described by a Euclidean metric obtained through the introduction of imaginary time. Consequently, quantum Euclidean wormholes appear as mathematical solutions of the Wheeler–DeWitt (WDW) equation and provide a useful framework for studying quantum gravitational phenomena, including quantum entanglement (Marugán, 1994; Mohsen, 2003; Davies,2005).

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[1] The null energy condition (NEC) is a fundamental constraint in general relativity stating that the energy density and pressure of matter, when measured along any light-like (null) trajectory, are non-negative. It ensures that light rays are focused, not repelled, by matter and is crucial for avoiding pathological physics like traversable wormholes.

[2] Exotic matter is a theoretical concept describing matter that possesses bizarre physical properties, such as negative mass or negative energy density. Negative mass is not anti-matter; it is a region where the energy of the universe is less than that of ordinary vacuum.

The WDW equation plays a pivotal role in quantum cosmology. Although its mathematical formulation remains incomplete, it is widely regarded as the fundamental equation of canonical quantum gravity, combining the principles of quantum mechanics and general relativity. It is often referred to as the "Schrödinger equation for the universe." It proposes that the total Hamiltonian operator, which operates on the wave functional of the universe, is zero. In this case, the wave functional is a function of the 3-dimensional spatial metric (geometry) and matter fields. The key aspect is that it does not contain an explicit time derivative. This gives rise to the well-known "problem of time" in quantum gravity, commonly referred to as the frozen formalism (Rotondo, 2022; Peres, 1999; Faizal, 2014). Since the WDW equation is independent of time, it can be used in both Lorentzian and Euclidean regimes.

Another fundamental concept in quantum cosmology is the Hartle–Hawking (HH) no-boundary proposal (Hartle et al., 1983; Hawking, 1994). Instead of assuming that the universe began from a singular point, as in the classical Big Bang model, the HH proposal suggests that the universe has no boundary in either space or time; it is finite yet without edges, much like the surface of the Earth (Hartle et al., 2008; Hartle et al., 2008). The proposal is based on quantum cosmology and employs Euclidean time, whereby the temporal coordinate behaves like a spatial dimension near the origin of the universe. Consequently, the initial Big Bang singularity is smoothed out. So instead of a sharp "start," the universe is like a rounded surface. Quantum Euclidean wormholes are obtained as solutions of the Wheeler–DeWitt equation subject to the Hartle–Hawking boundary conditions. Euclidean wormholes are expected to play an important role near the Planck epoch. Therefore, the study of Euclidean wormholes may provide valuable insights into the physics of the very early universe.
In this review, the mathematical formulations of both classical and quantum Euclidean wormholes are discussed in detail. Methods for estimating the throat size of a wormhole are illustrated through specific examples. Before presenting the mathematical framework, we briefly review the differences between Lorentzian and Euclidean spacetimes.

2. Euclidean and Lorentzian space-time

Lorentzian and Euclidean metrics represent two distinct geometric structures. A Lorentzian metric describes spacetime with one timelike and three spacelike dimensions (e.g. (-, +, +, +) or (+, -, -, -)). In this framework, the temporal and spatial dimensions are fundamentally different. Lorentzian spacetime possesses light-cone structures that determine causal relationships. The Lorentzian metric is used in general relativity to classify spacetime intervals as spacelike, timelike, or null (lightlike). In the Lorentzian regime, the speed of light in vacuum represents the maximum attainable speed, thereby preserving causality. The present-day expanding universe is described using Lorentzian spacetime (Yahalom, 2025; Yahalom, 2024).

In contrast, the Euclidean metric treats the temporal and spatial coordinates on an equal footing (+, +, +, +). In this framework, all four coordinates are treated equivalently and no maximum speed limit exists (in view of pure mathematics). Consequently, the notion of Lorentzian causality is absent. In view of physical concept, this may appear a debatable issue. The Euclidean metric may bypass the singularity at the early universe in some cases by replacing the singular moment of the "Big Bang"—a point of infinite density and curvature—with a smooth, non-singular, four-dimensional geometry, often described as a "quantum bounce" or a "no-boundary" proposal (Yahalom, 2025; Yahalom, 2024). As the universe approaches the Planck epoch, the temporal coordinate may be analytically continued from real (Lorentzian) time to imaginary (Euclidean) time, causing it to behave like an additional spatial dimension. This removes the singular boundary at which the classical spacetime description breaks down. (Zhang, 2015; Barbero et al., 1996).

The Euclidean time, τ and the Lorentzian time, t are connected via Wick rotation[3] $(\tau = it)$. For some specific cosmological models, it can be shown that the universe may transit from an early, singularity-free Euclidean phase to the current Lorentzian phase over time to achieve thermodynamic equilibrium (Yahalom, 2025; Yahalom, 2024). Accordingly, Euclidean metrics are widely employed as mathematical tools in quantum gravity to investigate phenomena occurring in the very early universe. During this highly energetic epoch, wormholes may form naturally and could influence the subsequent evolution of the universe. Consequently, wormholes may provide valuable insights into quantum gravity and the search for a unified theory of fundamental interactions (Hebecker et al., 2018; Guendelman, 2010; Pavlović et al., 2023). In the next section, the mathematical formulation of classical Euclidean wormholes is briefly reviewed.

3. Classical Euclidean wormhole

As discussed earlier, a wormhole generally consists of two "mouths" connected by a "throat," potentially linking distant regions of spacetime or different times. The throat of a wormhole is the narrowest cross-section that connects the two regions (or “mouths”). It is the geometric bottleneck of the wormhole and represents a minimal-area surface. A semi-wormhole is a theoretical construct in quantum gravity and string theory that represents a "half" or single-sided version of a Euclidean wormhole.

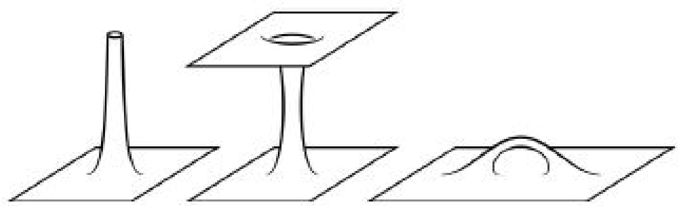

Figure 1: Different types of wormholes: (left) a semi-wormhole; (center) a wormhole connecting two asymptotically flat universes; and (right) a wormhole connecting two regions of the same universe. The narrow central region is called the throat, which connects the two asymptotically flat regions.

We consider a simple gravitational action consists of Ricci scalar, R minimally coupled with a scalar field, ϕ (Ruz et al, 2013; Rahaman et al., 2024),

$$S_c = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2\pi^2}\left(\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} + V(\phi)\right)\right] + \frac{1}{8\pi G}\int_{\Sigma} d^3x\,\sqrt{h}K \tag{1}$$

[3]Wick rotation is a technique, often used in physics. It converts Minkowski space-time (with real time) into Euclidean space (with imaginary time) by applying the transformation $\tau = it$. It is named after Gian Carlo Wick. This technique simplifies calculations in quantum field theory and statistical mechanics by making path integrals converge, transforming oscillating waves into exponential decay.

and examine whether it supports a wormhole solution. In the above action, $V(\phi)$ is some arbitrary potential of the scalar field, ϕ, $\sqrt{-g}$ is the invariant volume element determinant where g is the determinant of the spacetime metric $g_{\mu\nu}$ and $\phi_{,\alpha} = \frac{\partial\phi}{\partial x_\alpha}$. The first term represents the Einstein–Hilbert action minimally coupled to the scalar field. The second term is the Gibbons–Hawking–York boundary term (surface term), which cancels the total derivative arising from the Einstein–Hilbert action. The surface term includes h (determinant of the induced metric,h_{ij}) and K (trace of the extrinsic curvature,K_{ij}). Here, the energy-momentum tensor is given by (Ruz et al, 2013)

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\partial S_\phi}{\partial g_{\mu\nu}} = \phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left[\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} + V(\phi)\right] \tag{2}$$

with S_ϕ being the action for the matter field only. The field equations for such an action may be computed as (Ruz et al, 2013)

$$R_{\mu\nu} = 8\pi G\left[\phi_{,\mu}\phi_{,\nu} + g_{\mu\nu}V(\phi)\right] \tag{3}$$

Our objective is to determine whether these field equations admit wormhole-like solutions. As finding solution of the general field equations (3) is mathematically too difficult, we shall consider the Robertson-Walker (RW) minisuperspace for simplicity.

3.1 Gravitational action in RW minisuperspace

RW metric is defined by (Ruz et al, 2013),

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2}r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \tag{4}$$

where, $a(t)$ is the scale factor and the curvature parameter, $k = 0, \pm 1$, stands for the flat, closed and open models respectively. In this minisuperspace, Ricci scalar,

$$R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right)$$

Here, $\dot{a} = \frac{da}{dt}$ and $\ddot{a} = \frac{d^2a}{dt^2}$. Again, in RW minisuperspace, the trace of extrinsic curvature,

$$K = -3\frac{\dot{a}}{a}$$

and

$$\int d^4x\sqrt{-g} = -\int d^3x\sqrt{h}\int dt = Aa^3\int dt$$

where A is some constant. Thus, in RW minisuperspace, if we replace the respective curvature quantities and remove the total derivative term $(\ddot{a})$, comes from the Ricci scalar, R, it will cancel out with the 2-nd part of the action (1), i.e. the surface term. Now, action (1) reduces to (Ruz et al, 2013),

$$S_C = M\int\left[-\frac{1}{2}a\dot{a}^2 + k\frac{a}{2} + \frac{1}{M}\left(\frac{1}{2}\dot{\phi}^2 - V(\phi)\right)a^3\right]dt \tag{5}$$

where, $M = \frac{3\pi}{2G} = \frac{3}{2}\pi m_p^2$ (m_p is the reduced Planck mass, G is Newton's constant). The conjugate momenta corresponding to a and ϕ are,$P_a = -Ma\dot{a}$ and $P_\phi = a^3\dot{\phi}$. Thus the Hamiltonian corresponding to action (1) is,

$$H = \dot{a}P_a + \dot{\phi}P_\phi - L = -\frac{1}{2M}\frac{P_a^2}{a} + \frac{P_\phi^2}{2a^3} - \frac{M}{2}ka + a^3V(\phi)$$

where L is the corresponding Lagrangian. At this point we are able to compute the field equations from the expression (5) which corresponding to gravitational action (1) in RW minisuperspace.

3.2 Field equations

Expression (5) clearly indicates that the Lagrangian corresponding to gravitational action (1) in RW minisuperspace is a function of $a(t)$ and $\phi(t)$. Varying the Lagrangian with respect to $a(t)$, we obtain the following Friedmann Equation,

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{2}{M}\rho_\phi = \frac{2}{M}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \tag{6}$$

where, $\rho_\phi = \left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$. Again varying the Lagrangian with respect to scalar field, $\phi(t)$ we get the following scalar equation,

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + V_{,\phi} = 0 \tag{7}$$

where, $V_{,\phi} = \frac{dV}{d\phi}$.

Equations (6) and (7) correspond to the independent field equations in the Lorentzian spacetime with respect to the real time coordinate t. The remaining Einstein equations, namely the ((11)), ((22)), and ((33)) components, are not independent and therefore need not be considered separately. To find the corresponding Euclidean field equations, we shall apply Wick rotation $\tau = it$. Thus,

$$t = -i\tau, \quad i.e. \quad dt = -id\tau$$

Consequently, the time derivatives transform as follows:

$$\frac{d}{dt} = \frac{1}{-i}\frac{d}{d\tau} = i\frac{d}{d\tau}$$

Defining $a_{,\tau} = \frac{da}{d\tau}$, $\phi_{,\tau} = \frac{d\phi}{d\tau}$ and $\phi_{,\tau\tau} = \frac{d^2\phi}{dt^2}$ the field equations (6) and (7) become (Ruz et al, 2013)

$$\frac{a_{,\tau}^2}{a^2} = \frac{k}{a^2} + \frac{2}{M}\left(\frac{1}{2}\phi_{,\tau}^2 - V(\phi)\right) \tag{8}$$

$$\phi_{,\tau\tau} + 3\frac{a_{,\tau}}{a}\phi_{,\tau} = V_{,\phi} \tag{9}$$

In the next subsection we shall examine whether the above field equations satisfy wormhole conditions in view of existence of wormhole throat.

3.3 Wormhole solutions in view of consistent throat

A stable traversable wormhole should possess a static (non-evolving) throat (Morris et al., 1988), which is a constant-radius "tunnel" connecting two regions of space-time. As the throat acts as a surface of minimal area, separating the two asymptotically flat regions, $a_{,\tau} = \frac{da}{d\tau} = 0$ and $a(\tau) = a_{min}$ at wormhole throat. Here, we shall at first find out whether the above field equations (8) and (9) support existence of a wormhole throat, then claim that If such a throat exists, the corresponding spacetime may represent a classical Euclidean wormhole solution.
To find existence of throat, we shall consider $a(\tau)$ has the following form,

$$a^2 = l^2 + (\tau - \tau_0)^2 \tag{10}$$

where τ_0 and l are constants. Equation (10) suggests that, at $\tau = \tau_0$, $a = l$. This is the minimum value of a and at the same time it is non-zero. Further, $a \to \infty$ when, $\tau \to \infty$. Therefore, Eq. (10) describes two asymptotically flat regions connected by a finite throat. Taking derivative with respect to τ, equation (10) can be expressed as,

$$a_{,\tau}^2 = \frac{(\tau - \tau_0)^2}{a^2} = \frac{a^2 - l^2}{a^2} = 1 - \frac{l^2}{a^2}$$

It implies that, if $a_{,\tau}^2$ in equation (8) satisfies the above form, wormhole solution exists for the gravitational action (1) in RW minisuperspace. Specifically, this is a particular case.
In general, if

$$a_{,\tau}^2 = 1 - \frac{l^2}{a^n} \quad where\, n > 0 \tag{11}$$

where l and n are constants then it satisfies wormhole conditions with a non-zero throat [40].

Careful observation of equation (8) reveals that by choosing suitable form of the potential, $V(\phi)$ one can satisfy the condition given in equation (11). Now, if we consider a scalar field in the Lorentzian regime described by the following ansatz,

$$\dot{\phi} = \frac{l}{a^\alpha}$$

where α is a constant. Then, $\dot{\phi} = i\phi_{,\tau}$. In view of the above ansatz, one can solve equation (7) or (9) and found the following form of potential,

$$V = \frac{l^2(3-\alpha)}{2\alpha a^{2\alpha}} \tag{12}$$

The above form of potential suggests that, $\alpha \leq 3$, so that the potential remains positive definite. Now, substituting the above form of V in equation (8), we get

$$a_{,\tau}^2 = k - \frac{3l^2}{M\alpha a^{2(\alpha-1)}} \tag{13}$$

Thus following the ansatz $\dot{\phi} = \frac{l}{a^\alpha}$, field equations (8) and (9) reduces to equation (13). It is apparent that equation (13) is in the form of equation (11) for $k = 1$. In the process, the necessary condition for the wormhole which have a stable non-evolving throat is satisfied. So action (1) admits wormhole solutions under some specific conditions of its parameters.

Here, action (1) admits classical Euclidean wormhole solutions in RW minisuperspace only if, the form of $a(\tau)$ follows equation (10) and the ansatz $\dot{\phi} = \frac{l}{a^\alpha}$ is true. These conditions are highly restrictive. In reality, satisfaction of throat conditions alone does not guarantee a complete Euclidean wormhole geometry. For a wormhole to remain traversable without collapsing, the throat must satisfy several additional conditions, including the flare-out condition and the absence of horizons. (Cataldo et al., 2026). Again, traversable wormholes generally require "exotic matter" with negative energy density to prevent gravitational collapse (Cataldo et al., 2026). Global properties and asymptotic behavior must also be checked. Nevertheless, the existence of such wormhole solutions opens new directions from a theoretical perspective. Studying classical Euclidean wormholes is crucial in theoretical physics because they act as non-perturbative, semi-classical "saddles" in the quantum gravitational path integral. They allow us to look insight into quantum gravity. This also helps us to understand the black hole information paradox and get a probable solution (Dai, 2020; Stoica, 2018). Wormholes also help us to understand the nature of spacetime at very early universe.

One can also explore the other options too. Let us consider that the universe is made of pure radiation and do not contain any scalar field. With this approximation, the Euclidean field equation (8) reduces to the following form,

$$a_{,\tau}^2 = k - \frac{8\pi G}{3}\frac{\rho_r}{a^2}$$

where, ρ_r is a constant. Again, if we consider that the universe is made of pressure-less dust, equation (8) takes the form,

$$a_{,\tau}^2 = k - \frac{8\pi G}{3}\frac{\rho_m}{a}$$

where, ρ_m is a constant. Here, one can observe that both of the above two equation takes the form (13) for different values of α. Therefore, both radiation and pressure-less dust admit the classical Euclidean wormhole boundary condition. Consequently, whether a particular early-universe model admits classical Euclidean wormhole solutions can be determined using the procedure outlined above.
We now turn to quantum Euclidean wormholes. These are the solutions of the WDW equation derived from corresponding action. This topic is discussed in the following section.

4. Wheeler DeWitt Equation and boundary conditions

The WDW equation is actually quantum counterpart of the classical field equations. The corresponding quantum equation can be derived. for the action (1) in RW minisuperspace from the Hamiltonian constraint equation. The Hamiltonian for gravitational action (1) in RW minisuperspace is given in section 3.1. Accordingly, the Hamiltonian constraint equation for action (1) in RW minisuperspace can be expressed as (Ruz et al, 2013; Rahaman et al., 2024),

$$H = -\frac{1}{2M}\frac{P_a^2}{a} + \frac{P_\phi^2}{2a^3} - \frac{M}{2}ka + a^3V(\phi) = 0 \tag{14}$$

where P_a and P_ϕ are the corresponding momenta canonically conjugate to a and ϕ. Replacing the canonical momenta by the corresponding quantum operators yields the corresponding quantum equation. In canonical quantization,

$$p_a = -i\hbar\frac{\partial}{\partial a} \quad and \ p_\phi = -i\hbar\frac{\partial}{\partial\phi}$$

To account for the operator ordering ambiguity[4], we shall replace momenta corresponding to a and ϕ, i.e.,P_a and P_ϕ by

$$P_a^2 = -\frac{\hbar}{a^p}\left(\frac{\partial}{\partial a}\left(a^p\frac{\partial}{\partial a}\right)\right) = -\hbar^2\left(\frac{\partial^2}{\partial a^2} + \frac{p}{a}\frac{\partial}{\partial a}\right) \tag{15}$$

$$P_\phi^2 = -\frac{\hbar^2}{\phi^q}\left(\frac{\partial}{\partial\phi}\left(\phi^q\frac{\partial}{\partial\phi}\right)\right) = -\hbar^2\left(\frac{\partial^2}{\partial\phi^2} + \frac{q}{\phi}\frac{\partial}{\partial\phi}\right) \tag{16}$$

where, p and q are constants, called the operator ordering index. They represent the factor ordering ambiguities between the operators a and $\frac{\partial}{\partial a}$ as well as, ϕ and $\frac{\partial}{\partial\phi}$ respectively. The WDW equation for the present case can be expressed as,

$$\left[-\frac{\hbar^2}{2M}\left(\frac{\partial^2}{\partial a^2} + \frac{p}{a}\frac{\partial}{\partial a}\right) - \frac{M}{2}ka^2 - \frac{\hbar^2}{2a^2}\left(\frac{\partial^2}{\partial\phi^2} + \frac{q}{\phi}\frac{\partial}{\partial\phi}\right) + a^4V(\phi)\right]\psi = 0 \tag{17}$$

The values of p and q will depend upon the choice of minisuperspace. As WDW equation do not contain any time, it has the same form in both Lorentzian and Euclidean space-time. To solve the above equation, we have to apply suitable boundary conditions. In this review, we adopt the Hawking–Page boundary conditions (Hawking et al., 1990) which define Euclidean quantum wormholes as solutions to the WDW equation, which ensure the regular behaviour of the wave function (ψ) in the early universe. The Hawking–Page boundary conditions require that

[4]Operator ordering ambiguity is an important issue in quantum mechanics. It is related with different orderings of non-commuting operators (e.g., position$\hat{x}$ and momentum, $\hat{p}$)while converting an equation from classical to quantum domain. In classical domain, position and momenta commutes with each other. However, in quantum domain, their corresponding operators do not commute ($\hat{x}\hat{p} \neq \hat{p}\hat{x}$). This often leads to confusion while writing Hamiltonian constraint equation in quantum domain.

- The wave function, ψ must vanish asymptotically as $a \to \infty$, which indicates that the wormhole is asymptotically Euclidean.
- The wave function, ψ must remain regular as $a \to 0$.
- The wave function, ψ must remain finite throughout the minisuperspace.

The Eq. (17) include a general potential, $V(\phi)$. In practice, a specific scalar-field potential is usually chosen to facilitate analytical or numerical solutions of the WDW equation. Several representative examples are discussed in the following section.

5. Quantum Euclidean Wormholes

In this review, I do not discuss the detailed solution of Eq. (17) subject to the Hawking–Page boundary conditions. These calculations are lengthy and would distract from the main purpose of this review. Instead, only the main results are summarized. Readers interested in the detailed mathematical derivations are referred to the cited references.

In one of our earlier works, we investigated quantum Euclidean wormhole solutions for different potentials for the action (1), considering operator ordering parameters $p = 1$ and $q = 0$ (Ruz et al., 2013). We considered massless scalar field, i.e., $V(\phi) = 0$; constant potential, i.e., $V(\phi) = V_0$, where V_0 is constant; power law potentials of the form $V(\phi) = V_0\phi^n$, where n is some integer (Particularly, massive scalar field $V = V_0\phi^2$, quartic potential, $V = V_0\phi^4$, inverse power potential, i.e. $V = \frac{V_0}{\phi}$, inverse cubic potential, i.e. $V = \frac{V_0}{\phi^3}$, $V = \frac{V_0}{\phi^4}$ etc.); exponential potential like $V = V_0 e^{-\frac{\phi}{\lambda}}$, where λ is a constant. We found that not all scalar-field potentials admit quantum Euclidean wormhole solutions satisfying the Hawking–Page boundary conditions. Specifically, the potentials, $V(\phi) = 0, V_0, V_0\phi^2, \frac{V_0}{\phi}$ admit quantum Euclidean wormhole solutions for suitable choices of the model parameters.

Similar analyses have also been carried out by other authors considering both operator ordering indexes p and q and found quantum Euclidean wormholes exists for $V(\phi) = 0$, V_0, $f(a)$, $\frac{V_0}{\phi^n}$, $V_0 e^{-\frac{\phi}{\lambda}}$ for some specific values of p, q and other parameters (Rahaman et al., 2024). They also obtained quantum Euclidean wormholes for different type of perfect-fluid matter source (Rahaman et al., 2024). These studies demonstrate that, within the RW minisuperspace, certain classes of gravitational actions admit quantum Euclidean wormhole solutions under suitable parameter restrictions.

Several authors have investigated quantum Euclidean wormholes within curvature squared gravity (Modak et al., 2024) and for modified theory of gravity (Sarkar et al., 2018). They showed that the wormhole configuration supports transition of the universe to the power law inflation asymptotically. Another group of researchers found that the initial Euclidean wormhole accompanied by oscillation of the scale factor in Euclidean time, asymptotically evolves into an exponentially expanding universe at later stage (Dutta et al., 2023). It has also been shown that wormhole solution exists in (4 + 1)-dimensional Kaluza–Klein cosmology minimally coupled with scalar field (Biswas et al., 2021). In this model, the cosmological evolution associated with the wormhole shows an inflationary era away from the classical forbidden domain. Euclidean wormhole configurations were also be obtained in 4-dimensional Robertson Walker Euclidean background with Einstein Gauss-Bonnet dilaton interaction (Biswas et al., 2022). In this case, authors demonstrated transition of universe from Euclidean regime to Lorentzian time, after passing through a phase characterized by oscillating Euclidean wormholes.

Consequently, the study of quantum Euclidean wormholes has become increasingly important in modern theoretical physics. Recent developments involving Euclidean wormholes have been applied. to resolve problems associated with black-hole physics, topology change and studying non-perturbative quantum effects (García-Compeán et al., 2020; Belin, 2026; Paul, 2021). These recent developments are discussed in the following section.

6. Recent developments on Euclidean wormholes

Recent research in quantum gravity has utilized Euclidean wormhole geometries to address the black hole information paradox. A notable development is the discovery of replica wormholes—complexified geometries in the gravitational path integral that connect different copies of a black hole's spacetime. These geometric connections define an "island" of information inside the black hole that couples to the outgoing Hawking radiation, driving the generalized entropy to follow the unitary Page curve. Together, islands and replica wormholes explain how information escapes a black hole (Almheiri et al., 2020; Penington et al. 2022).

The theory of Euclidean wormholes has been extensively applied in Jackiw-Teitelboim (JT) gravity, a simplified, exactly solvable 2D theory of quantum gravity. It naturally emerges as an essential toy model in theoretical physics, providing key insights into the AdS/CFT correspondence, black hole thermodynamics, and quantum chaos. When evaluating the path integral over Euclidean geometries, spacetime structures known as "Euclidean wormholes" can form, connecting disconnected asymptotic boundaries. In the context of holography, these wormholes indicate that the gravity path integral is dual to an average over an ensemble of random microscopic theories. Crucially, connected replica wormholes resolve the black hole information paradox by reproducing the unitary Page curve through the island formula (Engelhardt et al., 2021; Yu et al., 2026).

Recent holographic interpretations of Euclidean wormholes show that they compute averaged partition functions in dual theories, implying an ensemble average rather than a single exact boundary theory. In the AdS/CFT correspondence, these geometric bridges encode non-perturbative quantum effects and spectral statistics of quantum chaos, providing a key mechanism for resolving the black hole information paradox by preserving unitarity in the gravitational path integral (Chandra, 2024).

In the process, Euclidean wormholes have become a central focus in quantum gravity and the AdS/CFT correspondence. Their recent interpretations and developments on the factorization puzzle and disorder averaging (Belin, 2026; Maloney et al., 2025), the black hole information paradox (Kirklin, 2022), cosmological connections (Ambjørn et al., 2012) etc. have enriched theoretical quantum gravity to a great extent. However, there are many doubts and ambiguities exists in this field. The next section deals with the draw-backs of Euclidean wormholes.

7. Unresolved issues with Euclidean wormholes

From their inception, Euclidean wormholes have been studied in the contexts of quantum gravity, quantum cosmology, black hole physics, and string theory. However, significant fundamental questions remain regarding their physical interpretation and mathematical consistency. The most serious issue is that the Euclidean gravitational action is not bounded from below (Ambjørn et al., 2012). Because the conformal mode of the metric allows the action to become arbitrarily negative, the path integral diverges. Due to this instability, there is still no universally accepted definition of the Euclidean gravitational path integral (Marolf et al., 2025). Whether Euclidean wormholes represent legitimate contributions to quantum gravity or are merely artifacts of an ill-defined formalism remains an open question. Furthermore, because these wormholes exist in Euclidean time rather than observable Lorentzian spacetime geometries, physicists continue to debate whether these mathematical solutions represent genuine quantum gravitational phenomena or simply equations without direct physical counterparts.

Another issue lies with the so-called α-parameters. In quantum gravity, α -parameters represent the dimensionless coupling constants that parametrize the statistical effects of Euclidean wormholes on the fundamental constants of nature. This proposal raises several conceptual problems. Modern experiments indicate that the fundamental constants are universal and highly stable. If Euclidean wormholes produce varying coupling constants, an explanation is needed for why only one specific set of values is observed. Furthermore, the probability distribution governing these α -parameters remains unknown, leaving the physical interpretation incomplete

In quantum field theory, partition functions associated with disconnected boundaries are expected to factorize. However, in holographic systems, bulk Euclidean wormholes connecting multiple boundaries naturally generate correlations between otherwise independent boundary theories. This discrepancy is known as the factorization problem, raising fundamental questions regarding the compatibility of spacetime wormholes with exact holographic descriptions of quantum gravity (Belin, 2026).

Euclidean wormholes suggest the possibility of altering spacetime topology through quantum processes. While classical general relativity assumes a fixed topology, quantum gravity permits fluctuations connecting distinct geometric structures. Recent developments in replica wormholes indicate that these geometries are crucial to resolving the black hole information paradox. Despite these theoretical breakthroughs, several unresolved questions remain regarding phenomenological identity, semi-classical limitations and absence of experimental observation. Because these phenomena occur at the Planck scale, they remain beyond the reach of current technology. Although indirect effects involving axions, cosmology, or quantum gravity have been proposed, no conclusive observational evidence currently supports the existence of Euclidean wormholes.

8. Conclusion

This review comprehensively discusses both classical and quantum Euclidean wormholes, providing deep insights into the non-perturbative structure of spacetime within quantum gravity. By connecting distant regions through spacetime "handles" in an imaginary time framework, these geometries expand our understanding of general relativity. Euclidean wormholes are influential in path integral formulations, contributing as saddle points in the sum over geometries. Their presence suggests that physical constants and coupling parameters are not strictly fixed, but may instead be influenced by topological fluctuations of the spacetime continuum.

In modern theoretical physics—particularly regarding black hole information and holography—these wormholes have regained prominence as potential tools for resolving long-standing paradoxes. Research in this area is evolving rapidly, with ongoing efforts focusing on several key domains: Path integral formulation, holography, string theory, cosmology and particle physics etc.

Ultimately, while Euclidean wormholes are not yet fully understood or experimentally testable, they remain a powerful theoretical mechanism. Their study continues to shape our pursuit of a complete theory of quantum gravity, pointing toward a far more intricate spacetime structure than previously imagined.

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