Volume 2, Issue 2, 2025

Intuitions & Insights

An Interdisciplinary Research Journal

ISSN: 3048-6793

A Class of Renormings of l^p with the Fixed Point Property

Gopal Dutta

Department of Mathematics, J.K. College, Purulia, West Bengal, India

Email: gopalpartha92@gmail.com

Abstract: One of the open problems in metric fixed point theory is: Does every equivalent renorming of $l^p, 1 < p < \infty$, have the fixed point property? In this paper, we give some classes of renormings of $l^p, 1 < p < \infty$, with the fixed point property. It is known that $(l^p, \|.\|_p), 1 < p < \infty$, has the fpp.

Keywords: Fixed point property, Nonexpansive map, Reflexive Banach space.

Corresponding author: G. Dutta

Received: 20.03.2025; **Accepted**: 25.07.2025; **Published**: 30.07.2025

1. Introduction

The most crucial result in metric fixed point theory is Banach contraction principle, given by S. Banach in 1922. In 1965, the discovery of a key fixed point theorem for nonexpansive mappings gave the basis for the development of consequent theory in the area of metric fixed point theory.

Definition 1.1. Let K be a closed bounded convex subset of a Banach space X. A mapping $T: K \to K$ is said to be nonexpansive if $\|Tx - Ty\| \leq \|x - y\|, \forall x, y \in K$.

We say that K has the fixed point property (fpp) if every nonexpansive mapping on K has a fixed point in K. Also, the Banach space X has the fpp if every closed bounded convex subset of X has the fpp. Generally, a Banach space X may not have the fpp. The following example shows that c_0 does not have the fpp.